

ELECTRICAL CIRCUITS REVIEW

by Bruce M. Fleischer

This appendix is recommended reading if you want a review of electronics' basic ideas. Why, you may ask, does a digital circuit designer need to know this analog “stuff”? You need it to understand the abilities and shortcomings of components available in the real world, which must be used in building circuits that will work in the real world.

A practical problem that motivates this appendix is the calculation of the rise and fall times of a logic circuit output driving one or more inputs of other logic circuits (see Section 3.6 of the text). That problem is really one of analog, not digital circuits. Therefore, much of this appendix consists of building up enough ideas from analog electronics to make a simple model for rise and fall times.

B.1 FUNDAMENTALS

B.1.1 Charge

In the Bohr theory of the atom (named after Niels Bohr, 1885–1962), *electrons* orbit a nucleus containing *neutrons* and *protons*. Attraction between the opposite charges of electrons and protons keeps atoms together. Particles with the same charge repel each other. You may wonder what attraction holds the nucleus together, with all those protons. The answer is that the physicists have it all under control, and be glad this isn't an appendix on particle physics.

electron
neutron
proton

The nearly equal numbers of electrons and protons in most objects, such as a piece of fur, cancel out, so electrons in a neighboring object, such as an amber rod, usually don't feel any overall attraction for the fur. In some atoms, however, the electrons are not as tightly held as in others, and there are situations in which some electrons come loose.

For example, if we rub the amber rod on the fur, some of the electrons from the fur end up on the amber. The fur is missing some electrons, while the amber has extras. You can see the attraction of these charges for each other; the hairs in the fur are pulled towards the rod. If the two are brought close enough, some electrons in the rod will jump back to the fur, making a spark. The ancient Greeks didn't know about the Bohr model of the atom, but they were familiar with fur and amber (which is just petrified tree sap); in fact, our word "electron" comes from the Greek for amber.

coulomb

Electric charge is measured in *coulombs*. An individual electron or proton has much much less than one coulomb of charge, -1.6×10^{-19} coulomb on an electron, $+1.6 \times 10^{-19}$ on a proton. The decision to call the electrons negative is simply a convention started by Benjamin Franklin (1706–1790). Nature says only that an electron's charge is the opposite of a proton's; there is nothing inherently negative about electrons, they could just as easily be called positive and protons negative.

B.1.2 Voltage

energy

The attraction of opposite charges means that *energy* is required to pull them apart, and that energy can be recovered when they come together again. In between, we say that the energy is kept as *potential energy*. When a system has potential energy, it has the potential to do work. *Work* here just means a more visible form of energy, not necessarily something we want done.

potential energy
work

The most familiar form of potential energy is gravitational potential energy. Because of the gravitational attraction between the earth and the objects on it, lifting objects gives them potential energy. When an object is dropped, that potential energy is rapidly converted into kinetic energy, which is even more rapidly converted into other forms of energy when the object hits something (e.g., sound, heat, kinetic energy of broken pieces). The more mass something has and the higher we lift it, the more potential energy it has. Fans of David Letterman (1947–) may remember just how much potential for mayhem a bowling ball has when lifted up six stories.

voltage, V
volt (V)

In electricity, the quantity analogous to height is *voltage*, usually indicated by the symbol V . Voltage is measured in *volts* (symbol: V), named after Alessandro Volta (1745–1827). (Notational example: $V_{\text{object}} = 2.5 \text{ V}$.) In digital circuits, we sometimes measure voltage in thousandths of a volt, or *millivolts* (mV). Increasing the voltage of one coulomb of charge by one volt gives it one *joule* (J) of electrical potential energy, named after James P. Joule (1818–1889).

millivolt (mV)

joule (J)

Because electrons have negative charge, we have to take electrons from a higher to a lower voltage to increase their potential energy. This is what happens

in a battery: electrons go into the positive terminal and come out of the negative terminal with more potential energy, which is converted to other forms of energy as the electron travels “up” through the circuit. There is a special unit for the amount of energy one electron gets when its potential is reduced by one volt, an *electron-volt* (symbol: eV), which equals 1.6×10^{-19} J.

electron-volt (eV)

There are several lessons to remember about voltage and potential energy. One is that nearly all everyday systems have tremendous reservoirs of potential energy of various kinds, and any give and take of potential represents a minuscule fraction of a system’s total potential energy.

As a simple example, shown in Figure B-1, get a fresh bowling ball and carry it up one story, giving it one bowling-ball-story (*bbs*) of gravitational potential energy. Then drop it back to ground level to let loose that 1 bbs of energy. Does the bowling ball now have no potential energy? Certainly not—if you stand in a ditch under it, it clearly has lots of potential!

bbs

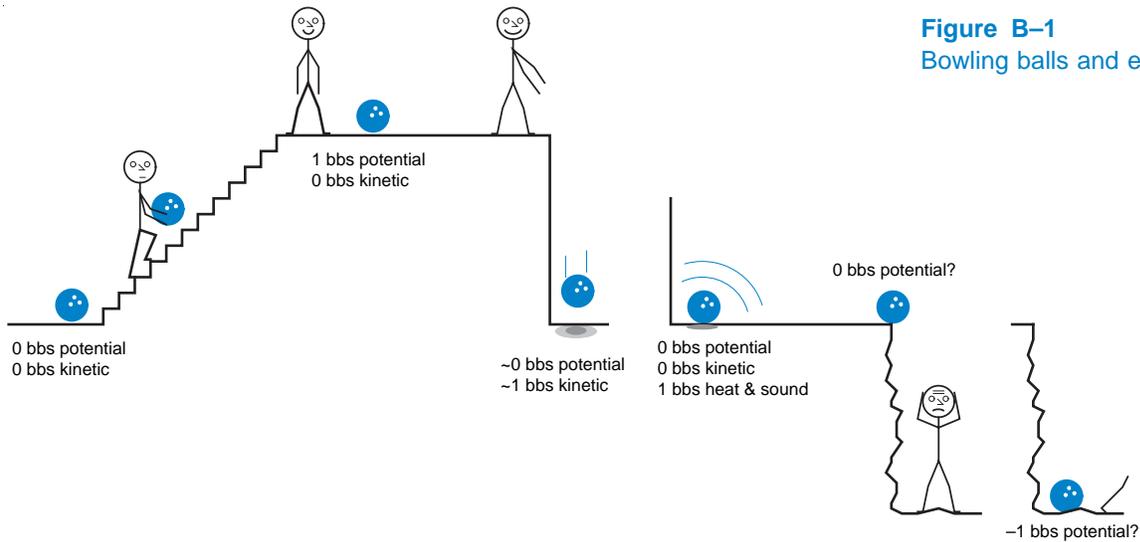


Figure B-1
Bowling balls and energy.

Stepping out of the way, let the ball drop into the ditch. Does it have -1 bbs of potential energy now? Clearly, some sort of convention needs to be set up here. We could try to find an absolute scale, say with 0 corresponding to ball at the center of the earth, but that would cause more problems than it would solve.

There are two workable conventions. The first is to consider potential energy as a purely relative term, and always speak of potential differences: the ball on the second floor has 1 bbs more potential than the one on the ground, and the one in the ditch is at -2 bbs relative to the second floor.

The second, shortcut convention uses the fact that many things sit at ground level. Thus, when we talk about the ball’s potential at a given spot, we can imply the comparison, “relative to ground level.” In this convention, “zero potential” just means that something is at ground level, not that it has absolutely no potential energy.

ground

In electrical terms, we can use the full form to describe the voltage difference, or voltage drop, between two points, or we can use the short form and talk about the voltage at a point. The short form always implies an unspoken comparison with a common reference point. In most circuits that common reference point is called *ground*, and may actually be connected to the ground through a water pipe, but it is not at zero volts in any absolute sense. (Strange problems, and sometimes fireworks, can result from connecting circuits whose “grounds” have a voltage difference between them.)

B.1.3 Current

current, I

ampere (A)
amp
second (s)

milliampere (mA)
microampere (μA)

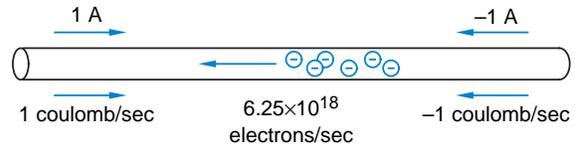
Electrical *current* has the same relation to charge as a river’s current does to water; it is the rate at which charge crosses a line cutting through part of the circuit. Current is usually indicated by the symbol *I* and is measured in *amperes*, or *amps* (symbol: A), named after A. M. Ampère (1775–1836). One ampere equals one coulomb per second (s). In digital circuits, we seldom have currents as high as one amp flowing into a single device. More often, we deal with currents of *milliamperes (mA)* and *microamperes (μA)*.

Like the flow of water, current has a size and direction. Usually, we decide on a direction beforehand, and then call current flowing that way positive and the other way negative. The one tricky thing about electrical current is that in electronic circuits, the protons stay put and the (negatively charged) electrons move around. If 6.25×10^{18} electrons per second flow through a wire from left to right, we can call that current -1 A flowing to the right or 1 A flowing to the left.

positive current

As shown in Figure B-2, the convention is that a certain current flows in the direction in which positive charge would move to make that current, which is the opposite of the direction in which electrons actually flow. To explicitly indicate the use of this convention, current is sometimes called *positive current*. If this seems confusing at first, blame it on Ben Franklin. We’re all too committed to this system to flip things around, so you might as well get used to it.

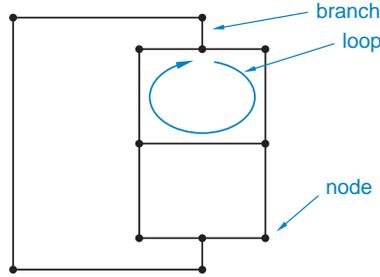
Figure B-2
Conventions for current flow.



B.1.4 Circuits

The schematic diagrams that we use to represent circuits on paper are similar to figures studied in graph theory, a topic in abstract mathematics. We can use some ideas from graph theory to make formal rules about how circuits work. Fortunately, the abstract ideas we need can be directly related to physical circuits; there isn’t any heavy math here.

Figure B-3
A graph.



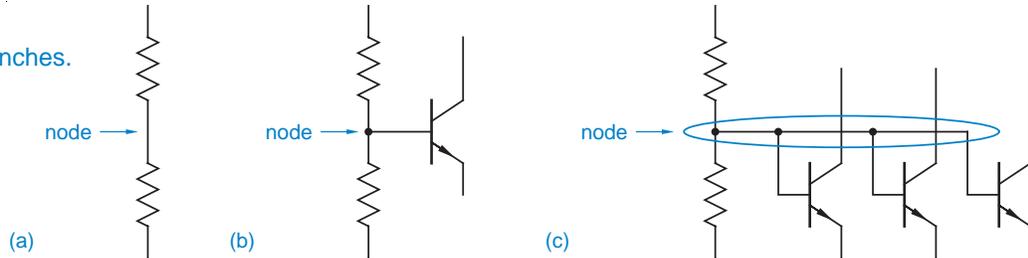
An example of a *graph* is shown in Figure B-3. It is made of points, called *nodes*, connected by lines, called *branches*. The only other abstract concept we need is a *loop*, which is just a path starting at some node, passing through a sequence of nodes and branches, and returning to the starting node. A direction is associated with each branch in the loop when we specify the loop's direction.

graph
node
branch
loop

The graph in Figure B-3 looks like a circuit diagram, albeit a boring one. We could use it to represent a circuit, though, with some rules for transforming schematics into graphs. It's simpler, however, just to refer to schematics as if they were graphs. When we do, there are two concepts we need to consider.

The first concept is the difference between nodes and connection dots. A node is any "point" in a circuit, as shown in Figure B-4. Part (a) indicates that there is a node between the resistors, although we wouldn't put a connection dot there. The points where we do put connection dots, as in (b), generally correspond to nodes. The circled part of the schematic in (c) would usually be considered to be a single node, unless the connecting wiring is so nonideal that it must be treated as additional components.

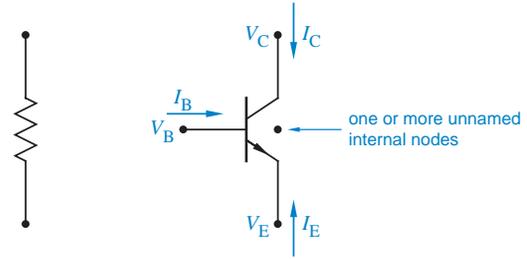
Figure B-4
Nodes and branches.



The second concept is how to treat components, as shown in Figure B-5. Two-lead components, such as resistors, are like single branches with a node at each end. Transistors and other larger components have at least one node per lead. Depending on how much detail we're interested in, we might imagine that there are one or more internal nodes also; that way we can give names to the voltages at and currents in each of the components' leads.

If a circuit diagram includes a node for ground, then the "single-ended" voltage at a node means the voltage difference from there to ground. The branches in the circuit diagram indicate physical paths through which current can flow, such as wires. Because current can't jump out of the circuit, the current flowing

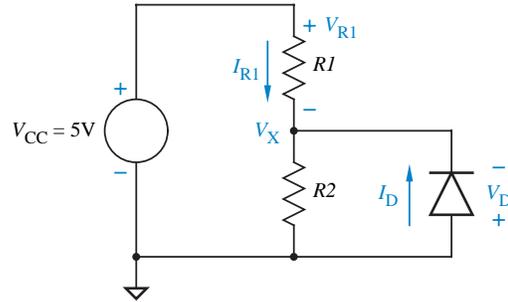
Figure B-5
Components.



in a branch is the same anywhere along the branch. In other words, there is a definite value of current in each branch of the circuit. There is also a voltage drop (a “differential” voltage) along each branch, the voltage difference between the nodes it connects. The one term you should avoid is the current at a node; always refer to the current through some branch of a circuit.

Once we’ve picked a direction for a branch, its current (or voltage drop) is a positive or negative number depending on whether the current flows (or voltage decreases) in the chosen direction. Figure B-6 shows part of a circuit diagram with several named voltages and currents. The + and – signs and the arrows show which polarity of voltage or current is called positive.

Figure B-6
A circuit diagram with explicit voltages and currents.



Kirchhoff's current law (KCL)
Kirchhoff's voltage law (KVL)

Using the terms defined above, we can write down two basic laws that apply to all circuits. They are called *Kirchhoff's current law (KCL)* and *Kirchhoff's voltage law (KVL)*, after Gustav Robert Kirchhoff (1824–1887).

KVL The sum of the voltage drops around any loop is zero.

KCL The sum of the currents into any node of a circuit is zero.

All these laws really say is that the business of assigning voltages to nodes, and currents to branches, is consistent.

KVL is like saying: if you start at any point in a building and follow any path (up and down stairs, elevators, and escalators) that comes back to the same point, the net number of stories you went up or down is zero.

One way of restating KCL is to say that nodes don't store charge; any current that comes into a node through one connection has to go out through another. Implicit in KCL is a rule about components: any current going into a

component through one lead must come out elsewhere. A resistor, for example, has two leads. Current can go through a resistor, but can't accumulate there.¹

Kirchhoff's laws may seem obvious. In a way, that's their point: to formalize the basic ideas we take as axioms. They're not proof that our picture makes sense, just a formal statement of the rules we use. In other words, they're a way of avoiding lame answers like "that's how electricity works." When we calculate series and parallel resistances in the next section and you ask "why can you make that step?" we can point to Kirchhoff's laws.

What makes Kirchhoff's laws true? That's how electricity works. This may not seem like much of an improvement, but at least we know exactly what we need to have faith in, which is about the most you can hope for in science.

B.2 RESISTORS AND EQUIVALENT CIRCUITS

B.2.1 Ohm's Law and Resistors

Take a chunk of any material and hook up two wires to it, to make a two-lead component. Call the voltage across the component V_{chunk} , and the positive current flowing through the component I_{chunk} . For most materials, the current in a component made in this way would be proportional to the voltage. The ratio $V_{\text{chunk}}/I_{\text{chunk}}$ is R , the component's *resistance*, and is measured in *ohms* (symbol: Ω), named after Georg Simon Ohm (1787–1854).

resistance, R
ohm
Ohm's law

The equation $V = I \cdot R$ is called *Ohm's law*. Either current or voltage can be thought of as the driving term. A resistance of one ohm means that one volt is required to get one amp to flow, or that a current of one amp produces a voltage drop of one volt. Because such a component can be described by giving its resistance, it is called a *resistor*. Most of the resistors used in real circuits have resistances of many ohms, so larger units are used for convenience (1 k Ω = 1000 Ω , 1 M Ω = 1000 k Ω). In circuit diagrams, a resistor is usually drawn together with a name and its resistance, often with the " Ω " omitted from the resistance (e.g., 470, 10k) since no other units would be appropriate.

resistor

If any number of resistors are connected in a network with two external terminals, the resulting object is also a resistor. That is, the total current flowing through the network from one terminal to the other is proportional to the voltage between the two terminals. Careful application of Kirchhoff's laws and some linear algebra can be used to calculate the resistance of any such network, but most of the circuits you'll work with don't need this level of math. Usually, a couple of simple rules about series and parallel combinations, given below, are enough. These rules can be derived from Kirchhoff's laws, but will be easier to

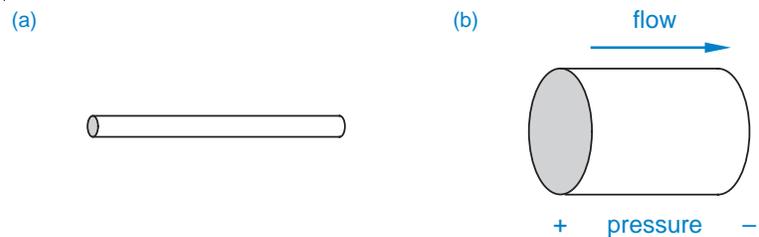
¹Those of you who are ahead of the game may protest this statement, because we can always force some charge onto any finite-sized component. The response is that to do so changes the voltage on that component, and the proper model includes a capacitance to ground from at least one interior node. When current through this capacitor is included, the terminal currents of the component sum to zero.

apply and will seem almost obvious given the right intuitive picture of current and resistance.

The bowling-ball analogy, used earlier to introduce the concept of voltage, suggests that current flowing in a resistor might be something like a bowling-ball waterfall. This is not the right intuitive picture; in fact, it is best to avoid picturing individual electrons at all when thinking about macroscopic currents.

Instead, water flowing in pipes provides a better analogy for electric current. Pressure represents voltage, current is current, and resistance is resistance (see Figure B-7. Fat pipes are the wires, and capillary tubes or pipes packed with porous material are resistors. The flow through such a resistor is proportional to the pressure imposed across it. To make the resistance greater, finer packing can be used, or the tube can be made narrower or longer. Coarser packing or a shorter or fatter piece of tubing reduces the resistance. The discussion to follow is given in electrical terms, but you can translate it to this “water in pipes” vocabulary if you want.

Figure B-7
Water-pipe analogy for resistance.



series resistance

The simplest resistor combination is two identical resistors in *series*, as shown in Figure B-8(a). The current is the same in the two resistors, so the total voltage drop is just double the drop across one. Therefore, the resistance (V/I ratio) of the combination is twice that of a single resistor, or $2R$. In the more general case in (b), the resistance of any number of resistors in series is the sum of the individual resistances:

$$R_{\text{series}} = R_1 + R_2 + \dots + R_n$$

If one of the resistors has a much larger value than the others, it will dominate the series resistance (a plugged pipe has about the same resistance regardless of whether the unplugged section is short or long).

Take another look at the $R + R$ series combination. If one end is grounded and the other is connected to a known voltage, as in Figure B-9(a), what’s the voltage at the node between the resistors? The answer is obvious: $V_{\text{out}} = V_{\text{CC}}/2$. This circuit is an example of a *voltage divider*. A voltage divider takes two

voltage divider

Figure B-8
Series resistance: (a) identical resistors; (b) different resistors.

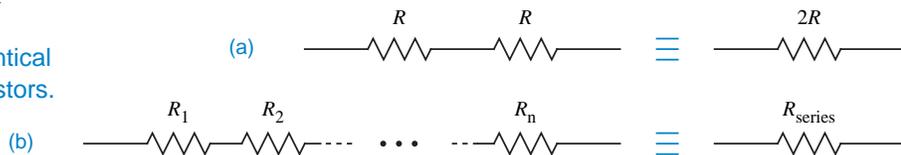
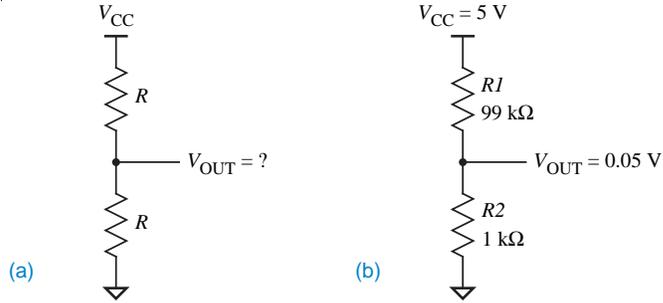


Figure B-9
Voltage dividers: (a) identical resistors; (b) different resistors.



voltages (one of which can be ground, but doesn't have to be) and produces a new voltage that is between them, dividing the original voltage difference into two smaller voltages.

Given the two input voltages, a divider's output voltage is set by the ratio of the resistors. It is halfway between the inputs if the resistors are equal, and is closer to one input if the resistor on that side is smaller. The divider in Figure B-9(b), for example, has an output voltage of 0.05 V. This result is easy to calculate, assuming that the current is the same through both resistors. If any current enters or leaves the divider at V_{out} , this assumption does not hold, and the output voltage is harder to see intuitively. We'll return to this point later.

When resistors are arranged to offer *parallel* paths for the current, the combination's resistance is less than that of the individual resistors. Once again, the simplest case uses two identical resistors, as in Figure B-10(a). Both resistors see the same voltage drop, but now the total current is the sum of the two currents. Therefore, the net resistance is $R/2$.

parallel resistance

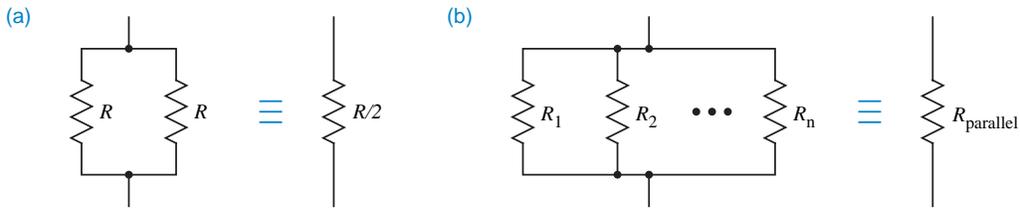


Figure B-10 Parallel resistance: (a) identical resistors; (b) different resistors.

The general case in Figure B-10(b) is easiest to work out if we think in terms of $1/R = I/V$, called the *conductance* or *admittance*. Because the total current through parallel resistors is just the sum of the individual currents (all resulting from the one applied voltage), the conductance of resistors in parallel can be written as the sum of individual conductances:

conductance
admittance

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

The symbol “||” is frequently used as shorthand for “in parallel”:

||

$$R_{\text{parallel}} = R_1 || R_2 || \dots || R_n$$

When there are only two resistors, the formula is often written in this form:

$$R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

If one resistor has a much smaller resistance (larger conductance) than the others, it dominates the parallel combination. Resistors in parallel can be viewed as a current divider, although they're seldom used that way.

Sometimes we find a network of resistors that is not a simple series or parallel combination, but that can be solved by successive applications of the series and parallel formulas above (see Exercise B.3). Some networks are just too complicated to analyze with these rules. As noted above, however, the resistance can always be found through more explicit use of Kirchhoff's laws (see Exercise B.4).

B.2.2 Power

Batteries and power supplies give energy to the electrons passing through. In most circuits, the electrons' energy is converted to heat as the electrons flow through the circuit back to the supply. Heat is produced any time that current flows through a voltage drop, so the electrons come out having less potential energy than when they went in.

power dissipation, P

watt

The *power dissipation*, P , is the rate at which electrical energy is converted to heat energy, and is given by the voltage (energy/charge) times the current (charge/s). The unit for power is the *watt* (symbol: W), named after James E. Watt (1736–1819); $1 \text{ W} = 1 \text{ V}\cdot\text{A} = 1 \text{ joule/s}$.

Most of the heat dissipated by TTL circuits comes from the circuits' internal resistors. Because the current through a resistor is proportional to the voltage across it, power goes as the square of the voltage or current. One line of algebra gets you from Ohm's law to $P = I^2 \cdot R = V^2 / R$.

B.2.3 Thévenin Equivalent

ideal voltage source

The symbol for an *ideal voltage source* is shown in Figure B-11. This hypothetical device creates a voltage difference between its terminals that is independent of the current through those terminals. Like irresistible forces and immovable objects, ideal sources are only abstractions, but are useful in describing the behavior of real sources, such as batteries. The voltage difference between a battery's terminals is nearly independent of the current taken from it, but if you pull enough current out of a battery, its voltage will drop noticeably.

Figure B-11
An ideal voltage source.

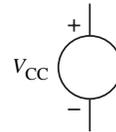
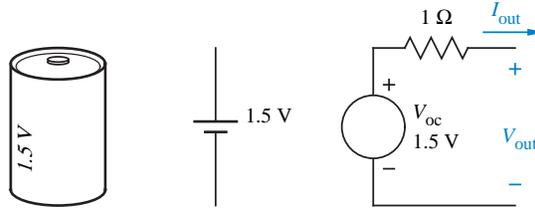


Figure B-12
Model for a real voltage source—a battery.



One way to describe this behavior of a real battery is to say that it acts like, or is modeled by, the circuit in Figure B-12, an ideal voltage source in series with a resistor. If the circuit doesn't have to supply any current, there is no drop across the resistor, and the source's voltage appears across the terminals (this is called the battery's *open-circuit voltage*, V_{oc}). If the battery's terminals are shorted together, the source's (unchanged) voltage is dropped entirely in the resistor, and the current through the short (the battery's *short-circuit current*) is $V_{oc}/R_{out} = (1.5 \text{ V})/(1 \ \Omega) = 1.5 \text{ A}$.

open-circuit voltage

short-circuit current

More completely, we can write a general formula relating the model's current and output voltage:

$$V_{out} = V_{oc} - I_{out} \cdot R_{out}$$

This linear model has two parameters, V_{oc} and R_{out} , that can be found from two measurements of V_{out} and I_{out} , such as the open-circuit voltage (V_{out} given $I_{out} = 0$) and the short-circuit current (I_{out} at which $V_{out} = 0$).²

Thévenin's theorem (named for Charles Leon Thévenin, 1857–1926) proves that any network of resistors and ideal voltage sources with only two terminals is equivalent to a single source in series with a resistor (see Figure B-13). The smaller circuit, called the *Thévenin equivalent*, is equivalent in the sense that it has the same I - V characteristics (the full circuit may dissipate more power than its Thévenin equivalent). As shown in the figure, the voltage and resistance used in the equivalent circuit are called V_{Th} and R_{Th} , the *Thévenin voltage* and *Thévenin resistance*. These parameters can be found from two measurements of the circuit's voltage and current, as described in the preceding paragraph.

Thévenin equivalent

Thévenin voltage

Thévenin resistance

To illustrate Thévenin equivalent circuits, let's take another look at the voltage divider in Figure B-9(a), with $V_{CC} = 5 \text{ V}$. With no load connected to V_{out} , the output voltage is obviously 2.5 V. What happens when we try to use that voltage, though? Figure B-14(a) shows a similar divider driving a load resistance (R_{load}) of 2 k Ω . It's not impossible to find V_{out} now, thinking of all the resistors as

²Real batteries or voltage sources are even less ideal than our model allows—the voltage is not necessarily a linear function of the current. One way to get around this is to use a linear model, but state that it only applies over a limited range. That is, we could say that the battery acts like the model in the figure, with $V_{oc} = 1.5 \text{ V}$ and $R = 1 \ \Omega$, for output currents between 0 and 100 mA. The measurements from which the parameters are found (or extracted) must be made within the stated range. Outside that range, the battery's behavior would have to be described by other parameter values, or perhaps even by a different model, such as one including flames or oozing chemicals.

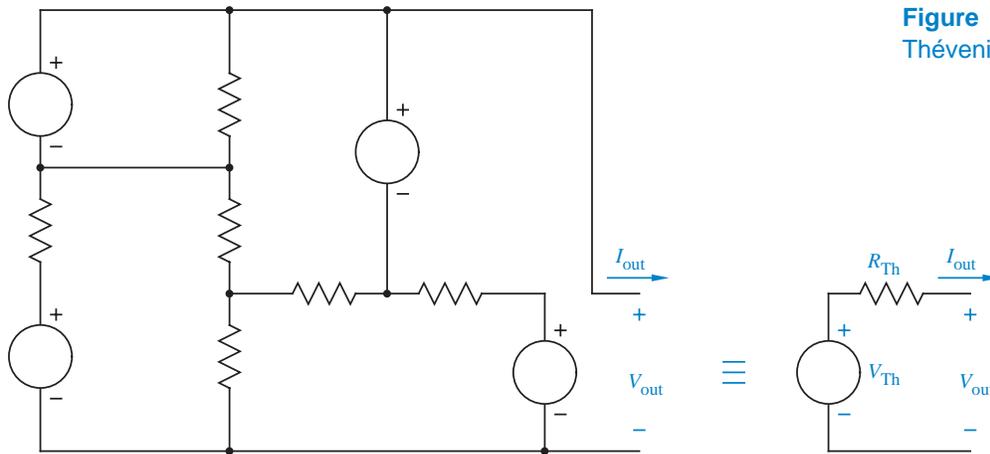


Figure B-13
Thévenin equivalent circuit.

one network, but it's easier to replace the source-plus-divider with its Thévenin equivalent, as shown in Figure B-14(b). Looking at the resulting divider, we see that V_{out} is 2.0 V. You might try putting in different load resistances and recalculating V_{out} . Which approach is easier when you do this?

Thévenin equivalents make calculations easier once we have them, but you may wonder where the foregoing values of V_{Th} and R_{Th} came from. A network's Thévenin voltage is just the network's open-circuit voltage, and is often pretty easy to figure out. One way to find the Thévenin resistance, which equals R_{out} , is to calculate the network's short-circuit current and then solve for R_{out} from that and the open-circuit voltage. This approach is a bit messy, but if you work through it, you'll get

$$R_{Th} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

At least you only have to do that once, and the answer is easy to remember because it equals $R_1 \parallel R_2$. Actually, that equality is no coincidence. It's a conse-

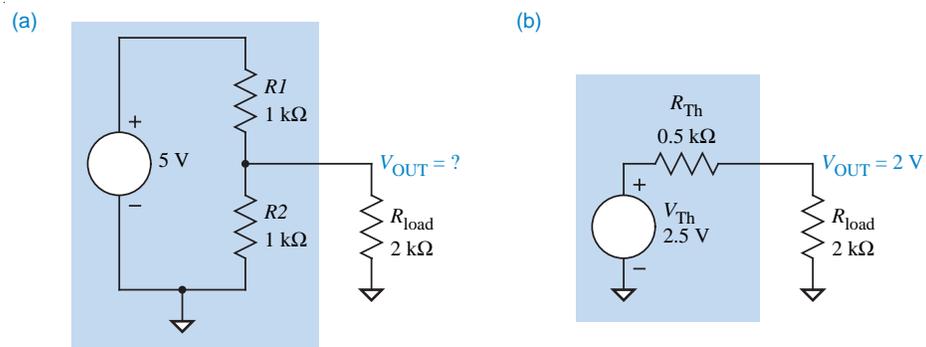


Figure B-14 Using the Thévenin equivalent: (a) a voltage divider under load; (b) Thévenin equivalent.

quence of a theorem that says: If you replace all voltage sources in a network with shorts (i.e., set them to 0 V sources) and open all current sources, the resistance of the resulting network equals the Thévenin resistance of the original network.

B.3 CAPACITORS

A *capacitor* is a component that stores charge. The simplest capacitor is made from a pair of parallel metal plates, as shown in Figure B-15 (it's called a *parallel-plate capacitor*). Let's say that we take some electrons out of one plate, leaving it positively charged, and put them on the other plate, making it negatively charged. It's not hard to see that there's now a voltage difference between the two plates.

capacitor
parallel-plate capacitor

Try grabbing an electron on the positive plate and moving it over to the negative plate. It is attracted to the positive charge behind it and repelled by the electrons in front, so you've got to push to move it. That push gives it potential energy, which means that its voltage is decreasing. The more charge there is on the plates, the more push is needed, and therefore the bigger the voltage difference between the plates. In a resistor, current and voltage are proportional, but in a capacitor, charge and voltage are proportional. The proportionality is usually expressed as

$$Q = C \cdot V$$

where Q represents the charge on the first plate (in coulombs), V is the voltage between the plates (positive if the first plate is at the higher voltage), and C is the *capacitance* of the capacitor, which is measured in *farads* (symbol: F), named after Michael Faraday (1791–1867).

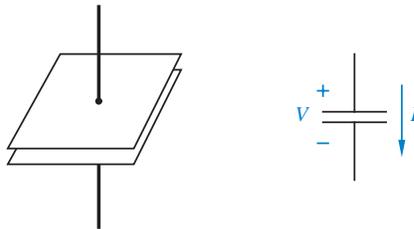
capacitance, C
farad

A capacitance of one farad means that putting +1 and -1 coulomb on the plates makes the voltage drop 1 V. You can also think of it the other way around: at a voltage of 1 V, a 1-farad capacitor holds 1 coulomb. Most capacitors are much smaller than 1 farad, so they're usually measured in *picofarads* (1 pF = 10^{-12} F) or *microfarads* (1 μ F = 10^{-6} F).

picofarad, pF
microfarad, μ F

The capacitance of a parallel-plate capacitor is directly related to the size and spacing of the plates. With the amount of charge fixed, pulling the plates farther apart takes work, and therefore increases the voltage drop. Said the other way, the farther apart the plates are, the less charge is accumulated per volt. Capacitance decreases for farther plates and increases for closer plates. Also, the

Figure B-15
A parallel-plate capacitor.



more area each plate has, the more charge can be stored, so a bigger area also means more capacitance.

One way to get a large capacitance in a small space is to use large sheets of very thin foil for the plates with a thin spacer to hold them apart, and roll everything up like a jellyroll. Larger capacitors, 1 μF and above, are often made this way. Smaller capacitors look more like the parallel-plate model, although their plates are usually separated by a solid material, such as ceramic or mica, rather than air. This material, called the *dielectric*, supports the metal plates and helps the capacitor hold more charge.

dielectric

Can other components, such as resistors or transistors, build up charge? If we stick electrons on a component, they'll make it harder to stick more on—and the component's voltage will drop. But remember, that's the voltage relative to ground. We can describe this effect by imagining that there are little capacitors between every part of every circuit and ground. The size of each capacitor tells us how much charge gets stored on that part of the circuit per volt.

As mentioned previously, changing the voltage across a capacitor requires changing the charge stored on each plate of the capacitor, which requires current through the capacitor. Each electron that goes in one lead of a capacitor can't come out the other lead because of the gap between the plates, but it does “push” one electron from the other plate out through the other lead. In a resistor, a fixed current creates a fixed voltage drop, but in a capacitor, a fixed current creates a steadily increasing voltage drop as charge accumulates on the plates. It's easy to put this relationship in quantitative terms. First, since the charge on one plate has to come through the attached lead, we can write

$$I = dQ/dt$$

We can take the derivative of charge equation $Q = C \cdot V$ and combine it with the preceding equation to derive the following equations:

$$dQ/dt = C \cdot dV/dt$$

$$I = C \cdot dV/dt$$

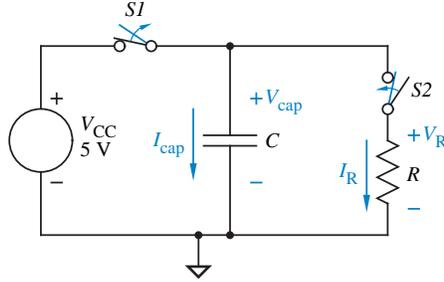
$$dV/dt = I/C$$

Thus, we can't instantly change the voltage across a capacitor, because that would require an infinite current. Capacitors are often explicitly placed at nodes (such as the power supply) where we want to hold a steady voltage. On the other hand, unavoidable stray circuit capacitances can prevent voltages from changing as fast as we might like them to, as we'll see in the next section.

B.4 RC CIRCUITS

Changing the voltage on a capacitor requires current. If that current must flow through a resistor, it requires a voltage across the resistor. If that voltage decreases as the capacitor charges, the current and the rate of charging will decrease exponentially with time.

Figure B-16
RC discharge.



For example, consider the circuit in Figure B-16. The circuit starts with switch $S1$ closed and switch $S2$ open, so the capacitor is charged to $V_{CC} = 5\text{ V}$ and no current can flow through the resistor. Now, suppose that we disconnect the voltage source by opening $S1$ and then start our stopwatches (or oscilloscopes) as we close $S2$. With $S2$ closed, notice that $V_R = V_{cap}$ and $I_R = -I_{cap}$ (if you agree, then you understand Kirchhoff's laws). We can write another two equations relating these variables, from the rules for resistors and capacitors:

$$\begin{aligned} I_R &= V_R / R \\ dV_{cap} / dt &= I_{cap} / C \end{aligned}$$

Substituting into the last equation we get

$$\begin{aligned} dV_{cap} / dt &= I_{cap} / C \\ &= -I_R / C \\ &= -V_{cap} / RC \end{aligned}$$

The solution to this equation is

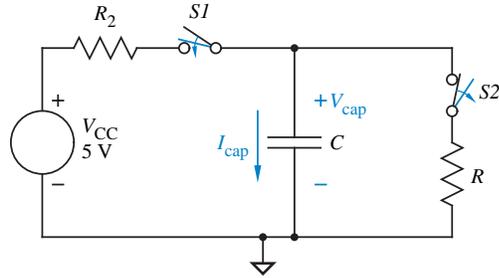
$$\begin{aligned} V_{cap}(t) &= V_{cap}(0) \cdot e^{-t/RC} \\ &= V_{CC} \cdot e^{-t/RC} \end{aligned}$$

In words, the capacitor's voltage decreases exponentially, with a time constant equal to RC . In order for the solution to make sense, the units for an RC product must be units of time, and this is indeed the case ($1\ \Omega \times 1\ \text{F} = 1\ \text{s}$, $1\ \text{M}\Omega \times 1\ \mu\text{F} = 1\ \text{s}$, $1\ \text{k}\Omega \times 1\ \text{pF} = 1\ \text{ns}$).

Now open $S2$, and close $S1$ again, as in Figure B-17, so V_{cap} goes back to 5 V . Does V_{cap} change instantaneously? Not in a real circuit, it doesn't. To make our figure a more accurate model, we need to include some resistance in series with the switch. This resistor, labeled R_2 in Figure B-17, can represent the resistance of the switch itself, or of a transistor serving as the switch, or perhaps of the voltage source. In any case, using the voltages and currents indicated in the figure, and ignoring R since no current can flow through it, we can now write

$$dV_{cap} / dt = (V_{CC} - V_{cap}) / R_2 C$$

Figure B-17
RC charge.



If the capacitor starts with no charge at $t = 0$, the solution to this equation is

$$V_{cap}(t) = V_{CC} \cdot (1 - e^{-t/R_2C})$$

The capacitor’s voltage approaches 5 V with an exponential time constant equal to R_2C . We can also write down the solution for an arbitrary initial voltage:

$$V_{cap}(t) = V_{CC} + (V_{cap}(0) - V_{CC}) \cdot e^{-t/R_2C}$$

This solution reduces to the previous one if $V_{cap}(0) = 0$. Actually, the simple RC discharge is also a special case of this solution, with $V_{CC} = 0$.

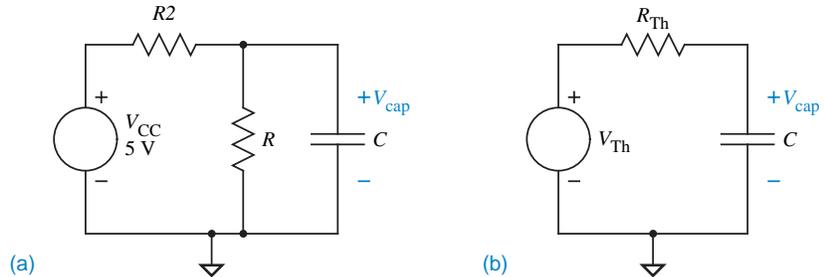
The natural next step is to close both switches at the same time. It is possible, using Kirchhoff’s laws, to write equations relating all the voltages and currents, and get a new differential equation for V_{cap} . There is, however, a much easier and much more intuitive way to figure out what happens, using the Thévenin equivalent of the divider. First, redraw the schematic so that the capacitor is separate, as in Figure B-18(a). Now the rest of the circuit can be replaced by its Thévenin equivalent, as in (b). The equivalent circuit has the same form as the charging circuit that we just analyzed, so the capacitor’s voltage is

$$V_{cap}(t) = V_{Th} + (V_{cap}(0) - V_{Th}) \cdot e^{-t/R_{Th}C}$$

As discussed above, a divider’s Thévenin voltage is just its open-circuit voltage, and its Thévenin resistance is the parallel combination of the two resistors.

More complicated problems can often be solved with this approach. Just move the capacitor off to one side, and draw a box around everything else. Since “everything else” is a two-terminal circuit, a Thévenin equivalent can be found, and the equation above can be used.

Figure B-18
Charging circuit: (a) redrawn;
(b) Thévenin equivalent.



EXERCISES

B.1 Over the lifetime of a D-size battery (2000 mA-hr capacity), how many electrons flow out of the negative terminal? Be more specific than “billions and billions.”

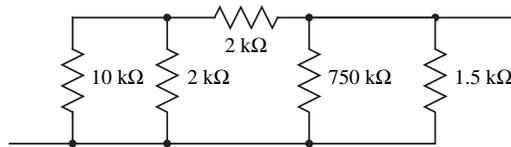
B.2 There is one electronic device that is something like a bowling-ball waterfall in the “electrons as bowling balls” analogy. That device is a cathode ray tube (CRT), used in televisions and computer displays. The electron gun in a CRT gives electrons a whole lot of potential energy with a high-voltage power supply, then converts all of that potential to kinetic energy as the electrons are fired at the face of the tube. The molecules of *phosphor*, a chemical coated on the inside of the tube, receive the electrons’ kinetic energy and radiate it away as visible light.

phosphor

Consider a computer display that accelerates the electrons through a voltage drop of 18,500 volts, has a beam current of $85 \mu\text{A}$, and scans the beam across a display with 640×864 pixels, 69 times each second. An electron’s mass is 9.1×10^{-31} kg. Ignoring relativistic effects, how fast do the electrons go? What fraction of the speed of light is this? About how many electrons are used to light up one pixel during one scan?

B.3 Calculate the resistance of the network in Figure XB.3.

Figure XB.3



B.4 What is the resistance of the resistor cube in Figure XB.4? You could put a 1-volt source across it, name the currents and voltage drops in every branch, use KCL to write equations for every node, use KVL to write equations for every loop, and solve them all to find the total current through the network. (*Hint:* There’s also a sneaky but easier way.)

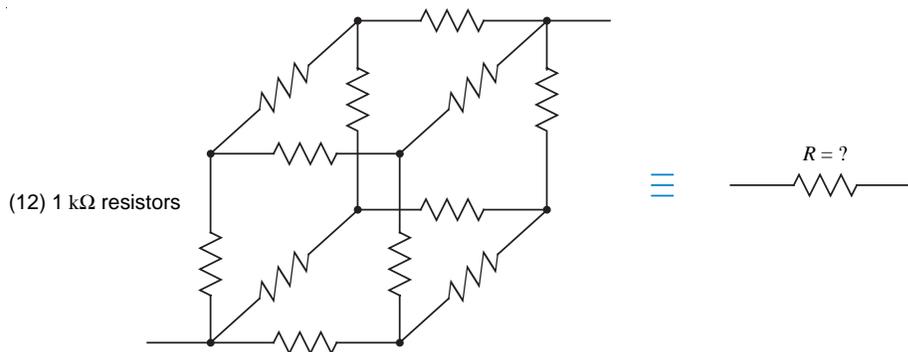


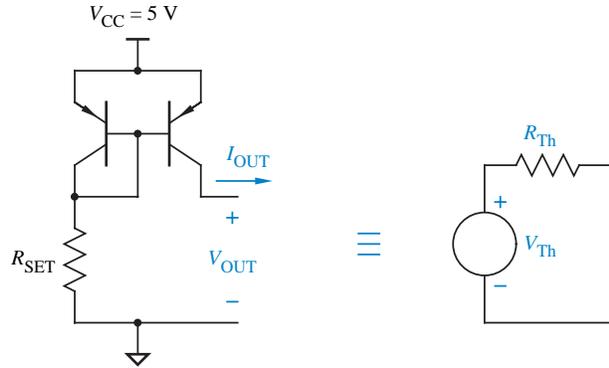
Figure XB.4

B.5 The circuit in Figure XB.5 is a good current source, in that its output current is nearly independent of its output voltage. When its output voltage is 0 V, it supplies

current mirror

1 mA of current, and at 3 V, it supplies 0.97 mA. Draw the Thévenin equivalent of this circuit (called a *current mirror*). (You don't have to understand transistor circuits to do this—you already have all the information you need.) Where does high-voltage source come from? (You *do* need to understand transistor circuits for this part.)

Figure XB.5



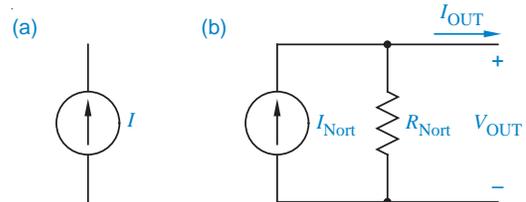
B.6 Thévenin equivalents are most natural for networks that we think of as almost ideal voltage sources. As Exercise B.5 showed, the Thévenin equivalent of a circuit that is close to an ideal current source can seem pretty strange.

Norton equivalent

The symbol for an ideal current source (current independent of voltage) is shown in Figure XB.6(a). Real current sources, of course, do not meet this ideal, although we can often model them with a *Norton equivalent* (named after E. L. Norton, 1898–), an ideal current source (I_{Nort}) in parallel with a resistance (R_{Nort}), as in (b). The Norton resistance makes the current vary somewhat with voltage. Mathematically, Thévenin and Norton equivalents are equally applicable; either can replace any network of resistors, voltage sources, and current sources.

Write I_{Nor} and R_{Nor} in terms of V_{Th} and R_{Th} . Draw the Norton equivalent of the current source in Exercise B.5.

Figure XB.6



- B.7 Show that the units of an RC product are time.
- B.8 In the circuit of Figure B-18, assume that $R_{Th} = 1\text{ k}\Omega$ and $C = 1\mu\text{F}$, and that V_{Th} is instantaneously changed from 0 V to 5 V. How long does it take for V_{cap} to reach 2.0 V?
- B.9 In the circuit of Figure B-18, assume that $R_{Th} = 50\text{ k}\Omega$ and $C = 1\mu\text{F}$, and that V_{Th} is instantaneously changed from 5 V to 0 V. How long does it take for V_{cap} to reach 0.8 V?